

# $B \rightarrow \pi l \nu$ Decay and $|V_{ub}|$

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## Abstract

$B \rightarrow \pi l \nu$  decay is studied in the effective theory of heavy quark with infinite mass limit. The leading order heavy flavor-spin independent universal wave functions which parametrize the relevant matrix elements are evaluated via light cone sum rule method in the effective theory. The important quark mixing matrix element  $|V_{ub}|$  is then extracted via  $B \rightarrow \pi l \nu$  decay mode.

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## I. INTRODUCTION

Weak decays of charmed and beautiful hadrons are quite favorable in particle physics because of their usage in determining fundamental parameters of the standard model and testing various theories and models. Among these heavy hadron decays the semileptonic decays  $B \rightarrow \pi l \nu$  and  $B \rightarrow \rho l \nu$  have been observed experimentally. These exclusive decays provide one of the main channels to determine the important CKM matrix element  $|V_{ub}|$ .

The difficulty in studying  $B \rightarrow \pi l \nu$  and  $B \rightarrow \rho l \nu$  decays mainly concerns the calculation of the relevant hadronic matrix elements of weak operators, or, equivalently, the corresponding form factors which contain nonperturbative contributions as well as perturbative ones and are beyond the power of pure QCD perturbation theory. Up to present these form factors are usually evaluated from lattice calculations, QCD sum rules and some hadronic models.

Sum rule method has been applied to  $B \rightarrow \pi(\rho) l \nu$  decay in the full QCD and provided reasonable results [1–3]. Since the meson B contains a single heavy quark, it is expected that its exclusive decays into light mesons may also be understood well in the effective theory of heavy quark, which explicitly demonstrates the heavy quark spin-flavor symmetry and its breaking effects can systematically be evaluated via the power of inverse heavy quark mass  $1/m_Q$ . The effective theory of heavy quark has been widely applied to heavy hadron systems, such as B decays into heavy hadrons via both exclusive and inclusive decay modes. There are two different versions of effective theory of heavy quark. One is the heavy quark effective theory (HQET), which generally decouples the "quark fields" and "antiquark fields" and treats one of them independently. This treatment is only valid when taking the heavy quark mass to be infinite. In the real world, mass of quark must be finite, thus one should keep in the effective Lagrangian both the effective quark and effective antiquark fields. Based on this consideration, a heavy quark effective field theory (HQEFT) [4–9] has been established and investigated with including the effects of the mixing terms between quark and antiquark fields. Its applications to the pair annihilation and creation have also been studied in the literature [10–12]. Though the HQEFT explicitly deviate from HQET from the next-to-leading order, these two formulations of effective theory trivially coincide with each other at the infinite heavy quark mass limit. In our knowledge the exclusive heavy to light (pseudoscalar) decay channels have been discussed in [13], where the matrix elements in the effective theory have been formulated, but the two leading order wave functions have not been calculated.

In this paper we focus on the calculation of the leading order wave functions of  $B \rightarrow \pi l \nu$  decay by using the light cone sum rule in the effective theory of heavy quark. As an important application,  $|V_{ub}|$  is extracted. In section 2, the heavy to light matrix element is represented by two heavy quark independent wave functions A and B. In section 3, we derive the light cone sum rules for the calculation of A and B. In section 4, we present the numerical results and extract  $|V_{ub}|$ . Our short summary is drawn in the last section.

## II. $B \rightarrow \pi L \nu$ DECAY MATRIX ELEMENT

The matrix elements responsible for  $B \rightarrow \pi l \nu$  decay is  $\langle \pi(p) | \bar{u} \gamma^\mu b | B \rangle$ , where b is the beautiful quark field in full QCD. It is generally parametrized by two form factors as follows,

$$\langle \pi(p) | \bar{u} \gamma^\mu b | B(p+q) \rangle = 2f_+(q^2)p^\mu + (f_+(q^2) + f_-(q^2))q^\mu. \quad (2.1)$$

In the effective theory of heavy quark, matrix elements can be analyzed order by order in powers of the inverse of the heavy quark mass  $1/m_Q$  and also be conveniently expressed by some heavy spin-flavor independent universal wave functions [5,8,9,13].

Here we adopt the following normalization of the matrix elements in full QCD and in the effective theory [5,8,9]:

$$\frac{1}{\sqrt{m_B}} \langle \pi(p) | \bar{u} \Gamma b | B \rangle = \frac{1}{\sqrt{\bar{\Lambda}_B}} \{ \langle \pi(p) | \bar{u} \Gamma Q_v^+ | B_v \rangle + O(1/m_b) \}, \quad (2.2)$$

where  $\bar{\Lambda}_B = m_B - m_b$ , and

$$\bar{\Lambda} = \lim_{m_Q \rightarrow \infty} \bar{\Lambda}_B$$

is the heavy flavor independent binding energy reflecting the effects of the light degrees of freedom in the heavy hadron.  $Q_v^+$  is the effective heavy quark field in effective theory.

Associate the heavy meson state with the spin wave function

$$\mathcal{M}_v = \sqrt{\bar{\Lambda}} \frac{1 + \not{v}}{2} \begin{cases} -\gamma_5 & \text{for pseudoscalar meson} \\ \not{\epsilon} & \text{for vector meson with polarization vector } \epsilon^\mu \end{cases} \quad (2.3)$$

we can analyze the matrix element in effective theory by carrying out the trace formula :

$$\langle \pi(p) | \bar{u} \Gamma Q_v^+ | B_v \rangle = -Tr[\pi(v, p) \Gamma \mathcal{M}_v] \quad (2.4)$$

with

$$\begin{aligned} \pi(v, p) &= \gamma^5 [A(v \cdot p, \mu) + \not{p} B(v \cdot p, \mu)], \\ \hat{p}^\mu &= \frac{p^\mu}{v \cdot p}. \end{aligned} \quad (2.5)$$

A and B are the leading order wave functions characterizing the heavy-to-light-pseudoscalar transition matrix elements in the effective theory. They are heavy quark mass independent, but are functions of the variable  $v \cdot p$  and the energy scale  $\mu$  as well. Nevertheless, since the discussion in the present paper is restricted within the tree level, we neglect the  $\mu$  dependence from now on.

Combining eqs. (2.1)-(2.5), one gets

$$f_\pm(q^2) = \frac{1}{\sqrt{m_b}} \sqrt{\frac{m_B \bar{\Lambda}}{m_b \bar{\Lambda}_B}} \{ A(v \cdot p) \pm B(v \cdot p) \frac{m_b}{v \cdot p} \} + \dots, \quad (2.6)$$

where the dots denote higher order  $1/m_Q$  contributions which will not be taken into account in the present paper. Note that we have used different variables for  $f_+$ ,  $f_-$  and  $A$ ,  $B$ . The relation between the variables  $v \cdot p$  and  $q^2$  is

$$y \equiv v \cdot p = \frac{m_B^2 + m_\pi^2 - q^2}{2m_B}. \quad (2.7)$$

### III. LIGHT CONE SUM RULE FOR $B \rightarrow \pi L \nu$

The QCD sum rule based on short distance expansion has been proved to be quite fruitful in solving a variety of hadron problems. Nevertheless, it is also well known that this method meets difficulties in the case of heavy to light transition because the coefficients of the subleading quark and quark-gluon condensate with the heavy quark mass terms grow faster than the perturbative contribution, which implies the breakdown of the short distance operator product expansion (OPE) in the heavy mass limit. Alternatively, it has been found that heavy to light decays can be well studied by light cone sum rule approach, in which the corresponding correlators are expanded near the light cone in terms of meson wave functions. In this way the nonperturbative contributions are embedded in the meson wave functions instead of the vacuum condensates in the short distance OPE sum rule. Though there are some differences in the techniques of calculation, the two sum rule methods are based on the same idea of quark-hadron duality and dispersion relation, and furthermore, they follow the same procedure in deriving form factors.

For  $B \rightarrow \pi l \nu$  decay, one may consider the vacuum-pion correlation function

$$F^\mu(p, q) = i \int d^4x e^{iq \cdot x} \langle \pi(p) | T \{ \bar{u}(x) \gamma^\mu b(x), \bar{b}(0) i \gamma^5 d(0) \} | 0 \rangle. \quad (3.1)$$

Here  $p$  and  $q$  are momenta carried by the pion and leptons. The B meson has momentum  $P_B = p + q$ . Inserting a complete set of states with B meson quantum numbers, we obtain the phenomenological representation

$$F^\mu(p, q)_{phen} = \frac{\langle \pi(p) | \bar{u} \gamma^\mu b | B \rangle \langle B | \bar{b} i \gamma^5 d | 0 \rangle}{m_B^2 - (p + q)^2} + \sum_H \frac{\langle \pi(p) | \bar{u} \gamma^\mu b | H \rangle \langle H | \bar{b} i \gamma^5 d | 0 \rangle}{m_H^2 - (p + q)^2}. \quad (3.2)$$

With the normalization relation in (2.2), the matrix elements in (3.2) can be expanded into the ones in effective theory of heavy quark in powers of  $1/m_b$ . When all higher  $1/m_b$  order contributions are neglected, (3.2) reduces straightforwardly into

$$2iF \frac{Av^\mu + B\hat{p}^\mu}{2\bar{\Lambda}_B - 2v \cdot k} + \int_{s_0}^{\infty} ds \frac{\rho(v \cdot p, s)}{s - 2v \cdot k} + \text{Subtractions}, \quad (3.3)$$

where  $k^\mu$  is the heavy hadron's residual momentum,  $k^\mu = P_B^\mu - m_b v^\mu$ . The first term in (3.3) is a pole contribution obtained by using (2.4) together with the parametrization

$$\langle 0 | \bar{q} \Gamma Q_v^+ | B_v \rangle = \frac{F}{2} \text{Tr}[\Gamma \mathcal{M}_v] \quad (3.4)$$

with  $F$  being the leading order decay constant of B meson in effective theory [8]. The second term in (3.3) is the higher resonance contributions given in the form of an integral over the physical spectral density  $\rho(v \cdot p, s)$ . Note that the Lorentz indices of the second term in (3.3) are not written explicitly but embedded in  $\rho(v \cdot p, s)$ .

On the other hand, the correlator can be calculated and expressed as the form of an integration over the theoretic spectral density  $\rho(v \cdot p, s)_{theory}$ , which equals to  $\rho(v \cdot p, s)$  under the assumption of quark-hadron duality. Namely, the correlator (3.1) can be written as

$$\int_0^\infty ds \frac{\rho(v \cdot p, s)}{s - 2v \cdot k} + \text{Subtractions}. \quad (3.5)$$

Equating (3.3) and (3.5) yields

$$2iF \frac{Av^\mu + B\hat{p}^\mu}{2\bar{\Lambda}_B - 2v \cdot k} = \int_0^{s_0} ds \frac{\rho(v \cdot p, s)}{s - 2v \cdot k} + \text{Subtractions}. \quad (3.6)$$

So the next step involves the calculation of (3.1) in the framework of effective theory of heavy quark. Substituting the heavy hadron states and heavy quark fields into the effective ones in the effective theory, and then performing the corresponding momentum shift  $P_B^\mu - m_b v^\mu = k^\mu$ , we obtain when neglecting higher  $1/m_Q$  order corrections

$$F^\mu(p, q) = i \int d^4x e^{i(q - m_b v) \cdot x} \langle \pi(p) | T \bar{u}(x) \gamma^\mu Q_v^+(x), \bar{Q}_v^+(0) i \gamma^5 d(0) | 0 \rangle. \quad (3.7)$$

In the light cone sum rule approach, one should contract the heavy quark fields and expand the correlator into a series in powers of the twist of light cone pion wave functions. These light cone wave functions provide an alternative treatment besides the vacuum condensates. They have been discussed in detail in many references [1,2,14]. Up to twist 4, the pion wave functions relevant to  $B \rightarrow \pi l \nu$  decay are defined as follows,

$$\begin{aligned} \langle \pi(p) | \bar{u}(x) \gamma^\mu \gamma^5 d(0) | 0 \rangle &= -ip^\mu f_\pi \int_0^1 du e^{iup \cdot x} [\phi_\pi(u) + x^2 g_1(u)] \\ &\quad + f_\pi (x^\mu - \frac{x^2 p^\mu}{x \cdot p}) \int_0^1 du e^{iup \cdot x} g_2(u), \\ \langle \pi(p) | \bar{u}(x) i \gamma^5 d(0) | 0 \rangle &= \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du e^{iup \cdot x} \phi_p(u), \\ \langle \pi(p) | \bar{u}(x) \sigma_{\mu\nu} \gamma^5 d(0) | 0 \rangle &= i(p_\mu x_\nu - p_\nu x_\mu) \frac{f_\pi m_\pi^2}{6(m_u + m_d)} \int_0^1 du e^{iup \cdot x} \phi_\sigma(u). \end{aligned} \quad (3.8)$$

$\phi_\pi$  is the leading twist 2 wave function.  $\phi_p$  and  $\phi_\sigma$  are twist 3 wave functions, while  $g_1$  and  $g_2$  are wave functions of twist 4.

Using the propagator  $\frac{1+\not{v}}{2} \int_0^\infty dt \delta(x - vt)$  for the contraction of the effective heavy quark fields, we get

$$\begin{aligned} F^\mu(y, \omega) &= -\frac{if_\pi}{2} \int_0^\infty dt \int_0^1 du e^{\frac{it\omega}{2}} e^{iyt(u-1)} \{v^\mu [tg_2(u) - i\mu_\pi \phi_p(u) - \frac{t}{6} \mu_\pi y \phi_\sigma(u)] \\ &\quad + \hat{p}^\mu y [-i\phi_\pi - it^2 g_1(u) - \frac{t}{y} g_2(u) + \frac{t}{6} \mu_\pi \phi_\sigma(u)]\} \end{aligned} \quad (3.9)$$

with  $\omega \equiv 2v \cdot k$  and  $\mu_\pi \equiv \frac{m_\pi^2}{(m_u + m_d)}$ .

In order to proceed, we perform a wick rotation of the  $t$  axis and then apply the Borel transformation  $\hat{B}_T^{(\omega)}$  to (3.9). The result is

$$\begin{aligned} \hat{B}_T^{(\omega)} F^\mu(y, \omega) &= -if_\pi \int_0^1 du e^{\frac{2y}{T}(u-1)} \{v^\mu [-\frac{2}{T} g_2(u) - \mu_\pi \phi_p(u) + \frac{1}{3T} \mu_\pi y \phi_\sigma(u)] \\ &\quad + \hat{p}^\mu y [-\phi_\pi(u) + \frac{4}{T^2} g_1(u) + \frac{2}{yT} g_2(u) - \frac{1}{3T} \mu_\pi \phi_\sigma(u)]\}. \end{aligned} \quad (3.10)$$

In deriving this equation we have used the feature of Borel transformation:

$$\hat{B}_T^{(\omega)} e^{\lambda\omega} = \delta(\lambda - \frac{1}{T}). \quad (3.11)$$

It is found that the spectral function used in sum rule can be obtained by performing a continuous double Borel transformation on the amplitude itself [15,16]. In order to get the spectral function  $\rho(y, s)$ , we now carry out the continuous Borel transformations as follows

$$\rho(y, s) = \hat{B}_{1/s}^{(-1/T)} \hat{B}_T^{(\omega)} F^\mu(y, \omega). \quad (3.12)$$

The result is

$$\begin{aligned} \rho(y, s) = & -\frac{if_\pi}{2y} \{v^\mu [\frac{1}{y} \frac{\partial}{\partial u} g_2(u) - \mu_\pi \phi_p(u) - \frac{\mu_\pi}{6} \frac{\partial}{\partial u} \phi_\sigma(u)] + \hat{p}^\mu y [-\phi_\pi(u) + \frac{1}{y^2} \frac{\partial^2}{\partial u^2} g_1(u) \\ & - \frac{1}{y^2} \frac{\partial}{\partial u} g_2(u) + \frac{\mu_\pi}{6y} \frac{\partial}{\partial u} \phi_\sigma(u)]\}_{u=1-\frac{s}{2y}}. \end{aligned} \quad (3.13)$$

In the derivation of (3.13),  $\frac{1}{T}$  has been first expressed as a derivative of the exponent in (3.10) over  $u$ , and then the method of integration by parts over  $u$  has been used.

(3.6) and (3.13) immediately yield:

$$\begin{aligned} A(y) = & -\frac{f_\pi}{4F} \int_0^{s_0} ds e^{\frac{2\bar{\Lambda}_B - s}{T}} [\frac{1}{y^2} \frac{\partial}{\partial u} g_2(u) - \frac{\mu_\pi}{y} \phi_p(u) - \frac{\mu_\pi}{6y} \frac{\partial}{\partial u} \phi_\sigma(u)]_{u=1-\frac{s}{2y}}, \\ B(y) = & -\frac{f_\pi}{4F} \int_0^{s_0} ds e^{\frac{2\bar{\Lambda}_B - s}{T}} [-\phi_\pi(u) + \frac{1}{y^2} \frac{\partial^2}{\partial u^2} g_1(u) - \frac{1}{y^2} \frac{\partial}{\partial u} g_2(u) + \frac{\mu_\pi}{6y} \frac{\partial}{\partial u} \phi_\sigma(u)]_{u=1-\frac{s}{2y}}, \end{aligned} \quad (3.14)$$

#### IV. NUMERICAL RESULTS

For the light cone wave functions appearing in the sum rules (3.14), we take [2,14,17]

$$\begin{aligned} \phi_\pi(u) = & 6u(1-u) \{1 + \frac{3}{2}a_2[5(2u-1)^2 - 1] + \frac{15}{8}a_4[21(2u-1)^4 - 14(2u-1)^2 + 1]\}, \\ \phi_p(u) = & 1 + \frac{1}{2}B_2[3(2u-1)^2 - 1] + \frac{1}{8}B_4[35(2u-1)^4 - 30(2u-1)^2 + 3], \\ \phi_\sigma(u) = & 6u(1-u) \{1 + \frac{3}{2}C_2[5(2u-1)^2 - 1] + \frac{15}{8}C_4[21(2u-1)^4 - 14(2u-1)^2 + 1]\}, \\ g_1(u) = & \frac{5}{2}\delta^2 u^2(1-u)^2 + \frac{1}{2}\epsilon\delta^2[u(1-u)(2+13u(1-u)+10u^3\log u(2-3u+\frac{6}{5}u^2) \\ & + 10(1-u)^3\log((1-u)(2-3(1-u)+\frac{6}{5}(1-u)^2))], \\ g_2(u) = & \frac{10}{3}\delta^2 u(1-u)(2u-1). \end{aligned} \quad (4.1)$$

The asymptotic form of these functions and the scale dependence are given by perturbative QCD [18,19].

For the convenience of comparison, we use the same values for the parameters as in [2,14],

$$\begin{aligned}
a_2(\mu_b) &= 0.35, \quad a_4(\mu_b) = 0.18, \quad B_2(\mu_b) = 0.29, \quad B_4(\mu_b) = 0.58, \\
C_2(\mu_b) &= 0.059, \quad C_4(\mu_b) = 0.034, \quad \delta^2(\mu_b) = 0.17\text{GeV}^2, \quad \epsilon(\mu_b) = 0.36.
\end{aligned}
\tag{4.2}$$

$\mu_b$  is the appropriate scale set by the typical virtuality of the beautiful quark,

$$\sqrt{m_B^2 - m_b^2} \approx 2.4\text{GeV}. \tag{4.3}$$

Besides all these parameters the numerical analysis of the sum rules (3.14) needs also the hadron quantities  $\mu_\pi$ ,  $f_\pi$ ,  $\bar{\Lambda}_B$  and  $F$ . These quantities have been studied via sum rules and other approaches by several groups. With the values  $\mu_\pi = 2.02\text{GeV}$ ,  $f_\pi = 0.132\text{GeV}$  [2,14],  $\bar{\Lambda}_B = 0.53\text{GeV}$  and  $F = 0.30\text{GeV}^{3/2}$  [8], we get from eqs.(3.14) the results for A and B given in the figures Fig.1-4. In these figures A and B are shown as functions of  $T$  and  $y = v \cdot p$ . We are mainly interested in the range of  $T = 2.0 \pm 1.0\text{GeV}$ , where both the twist 4 corrections and the contributions from excited and continuum states do not exceed 30%. It is seen that the curves in Fig.1 and Fig.2 are quite stable in this range for the threshold energy  $s_0 = 2.3 \pm 0.6\text{GeV}$ . In Fig.3 and Fig.4, A and B become rather stable with respect to the variation of  $y = v \cdot p$  when  $y > 1.5\text{GeV}$ . However, they become unstable at small  $y$ , which corresponds to large momentum transfer  $q^2$ . This is in expectation because the light cone expansion and the sum rule method would break down as  $q^2$  approaches near  $m_b^2$  [2].

We also derive  $f_+(q^2)$  and  $f_-(q^2)$  from  $A(v \cdot p)$  and  $B(v \cdot p)$  by using the relations in (2.6) and the beautiful quark mass  $m_b = m_B - \bar{\Lambda}_B = 4.75\text{GeV}$ . The results are shown in Fig.5-6. It is readily seen that when the momentum transfer  $q^2$  grows large (e.g. over  $16\text{GeV}^2$  for the curve of  $s_0 = 2.3\text{GeV}$  in Fig.5-6), the values of  $f_+$  and  $f_-$  derived from sum rules become rather unstable and should not be trusted.

In order to predict the decay width and  $|V_{ub}|$ , one should have knowledge on the behavior of form factors in the whole kinematically accessible region. Now for large momentum transfer we have the single pole approximation [2]

$$f_+(q^2) = \frac{f_{B^*} g_{B^* B \pi}}{2m_{B^*}(1 - q^2/m_{B^*}^2)}. \tag{4.4}$$

The couplings  $f_{B^*}$  and  $g_{B^* B \pi}$  have been studied in previous papers. Here we would use  $m_{B^*} = 5.325\text{GeV}$ ,  $f_{B^*} = 0.16 \pm 0.03\text{GeV}$  and  $g_{B^* B \pi} = 29 \pm 3$  [2].

Next we write  $f_+(q^2)$  as

$$f_+(q^2) = \frac{f_+(0)}{1 - aq^2/m_B^2 + bq^4/m_B^4} \tag{4.5}$$

and fit the parameters  $a$  and  $b$  by using the sum rules and eq. (4.4). For the threshold  $s_0 = 2.3\text{GeV}$ , we choose proper  $a$  and  $b$  to make (4.5) approach the sum rule results at  $q^2 < 15\text{GeV}^2$  but compatible with eq. (4.4) at  $q^2 > 15\text{GeV}^2$ . Our favorable parameters are

$$a = 1.31, \quad b = 0.35, \quad f_+(0) = 0.35. \tag{4.6}$$

The values of  $f_+(q^2)$  at  $T = 2.0\text{GeV}$  calculated from (4.4), (4.5) and the light cone sum rules are shown in Fig.7. It is found that the single pole model extrapolation matches quite well with the direct estimation from our light cone sum rules (3.14) at intermediate

momentum transfer around  $q^2 = 15\text{GeV}^2$ . This implies that our discription of  $f_+(q^2)$  by (4.4) together with the sum rules (3.14) (but in different applicable regions) is self-consistent.

For the lepton  $l = e$  or  $\mu$ , the lepton mass  $m_l$  may be safely neglected, and the decay width of  $B \rightarrow \pi l \nu$  has the distribution on momentum transfer  $q^2$  as follows

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} (E_\pi^2 - m_\pi^2)^{3/2} [f_+(q^2)]^2. \quad (4.7)$$

Here  $E_\pi = y = (m_B^2 + m_\pi^2 - q^2)/(2m_B)$  is the pion energy in the B meson rest frame.

With the pion mass  $m_\pi = 0.14\text{GeV}$  and the parametrizations of (4.5), we obtain the integrated width

$$\Gamma(B \rightarrow \pi l \nu) = (10.2 \pm 1.5) |V_{ub}|^2 \text{ps}^{-1}. \quad (4.8)$$

The error in eq.(4.8) results from the variation of the threshold energy in  $s_0 = 1.7 - 2.9\text{GeV}$ .

From the branching fraction measured by CLEO collaboration [20],  $\text{Br}(B^0 \rightarrow \pi^- l^+ \nu_l) = (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}$  and the world average of the  $B^0$  lifetime [21],  $\tau_{B^0} = 1.56 \pm 0.06 \text{ ps}$ , one has [2]

$$\Gamma(B^0 \rightarrow \pi^- l^+ \nu_l) = (1.15 \pm 0.35) \times 10^{-4} \text{ps}^{-1}. \quad (4.9)$$

Comparison of (4.8) and (4.9) yields

$$|V_{ub}| = (3.4 \pm 0.5 \pm 0.3) \times 10^{-3}, \quad (4.10)$$

where the first (second) error corresponds to the experimental (theoretical) uncertainty. Here the theoretical uncertainty is mainly considered from the threshold effects. In general, higher order contributions need to be included for all the relevant parameters. It was noticed that the two-loop QCD perturbative correction may be significant for an accurate determination of B meson decay constants [8]. In particular, it may enlarge the constant F by about 25%, and increase  $\bar{\Lambda}$  at the same time. These effects evidently worsen the accuracy of our extraction of  $|V_{ub}|$ . By taking into account this uncertainty, we arrive at the following result

$$|V_{ub}| = (3.4 \pm 0.5 \pm 0.5) \times 10^{-3}, \quad (4.11)$$

This estimate is in good agreement with that derived from full QCD calculation [2]:

$$\begin{aligned} |V_{ub}| &= (3.9 \pm 0.6 \pm 0.6) \times 10^{-3} \quad (\text{via } B \rightarrow \pi l \nu), \\ |V_{ub}| &= (3.4 \pm 0.6 \pm 0.5) \times 10^{-3} \quad (\text{via } B \rightarrow \rho l \nu). \end{aligned} \quad (4.12)$$

Furthermore, the value of  $|V_{ub}|$  obtained in eq.(4.11) is also close to the one given by CLEO [22],

$$|V_{ub}| = (3.25 \pm 0.14_{-0.29}^{+0.21} \pm 0.55) \times 10^{-3}, \quad (4.13)$$

which is a combined result from the analyses based on different models and treatments on  $B \rightarrow \pi(\rho) l \nu$  transitions.



## V. SUMMARY

In this paper we have studied  $B \rightarrow \pi l \nu$  decay by using the light cone sum rule approach within the framework of effective theory for heavy quark. Two leading order wave functions in the effective theory with infinite mass limit have been calculated. The important CKM matrix element  $|V_{ub}|$  has been extracted and its value has been found to be

$$|V_{ub}| = (3.4 \pm 0.5 \pm 0.5) \times 10^{-3}. \quad (5.1)$$

It has been seen that the value of  $|V_{ub}|$  extracted from the leading order heavy quark expansion coincides well with that extracted from the full QCD calculation, which shows the reliability of the heavy quark expansion and the power of light cone sum rule approach in studying heavy to light exclusive decays. Working out  $1/m_Q$  contributions should be interesting, and it is expected to cast more light on the treatment of heavy to light decays by applying for the effective theory of heavy quark.

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# FIGURES

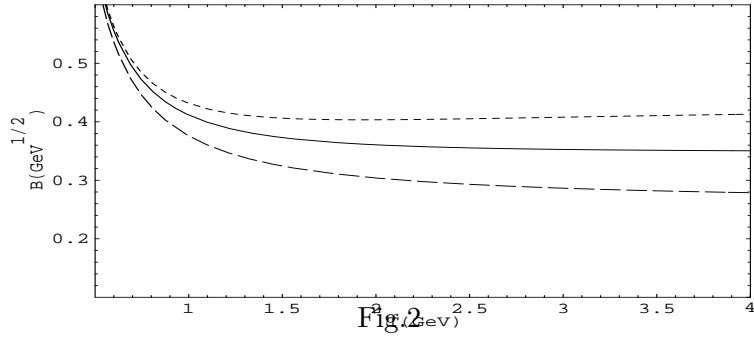
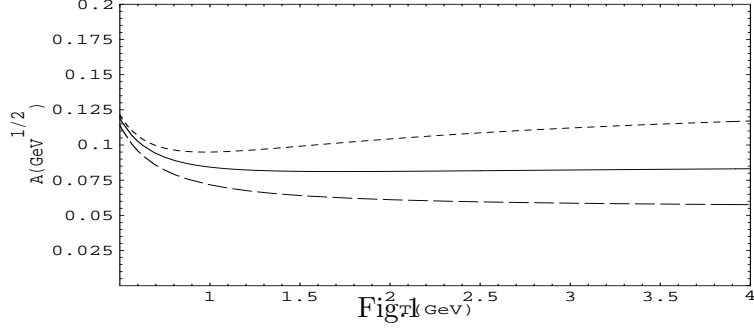


Fig.1-2. Variation of A and B with the Borel parameter T for different values of the continuum threshold  $s_0$ . The dashed, solid and dotted curves correspond to  $s_0=1.7, 2.3$  and  $2.9$  GeV respectively.  $y = v \cdot p = 2.64$  GeV is fixed, which corresponds to  $q^2 = 0 \text{ GeV}^2$ .

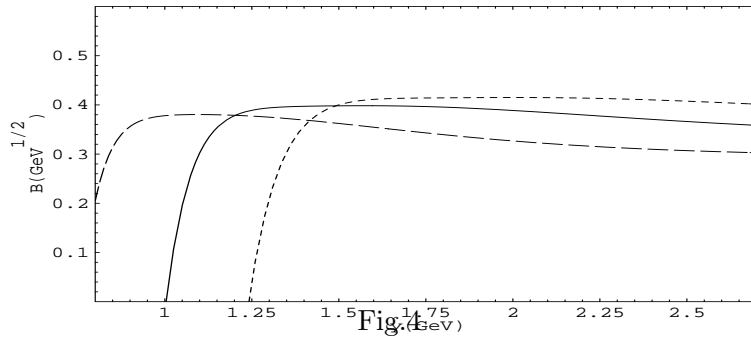
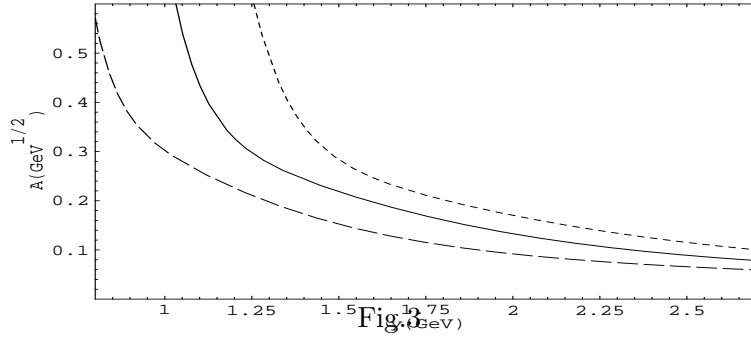


Fig.3-4. A and B as functions of  $y = v \cdot p$  for different values of the continuum threshold  $s_0$ . The dashed, solid and dotted curves correspond to  $s_0 = 1.7, 2.3$  and  $2.9$  GeV respectively.  $T = 2.0$  GeV is fixed.

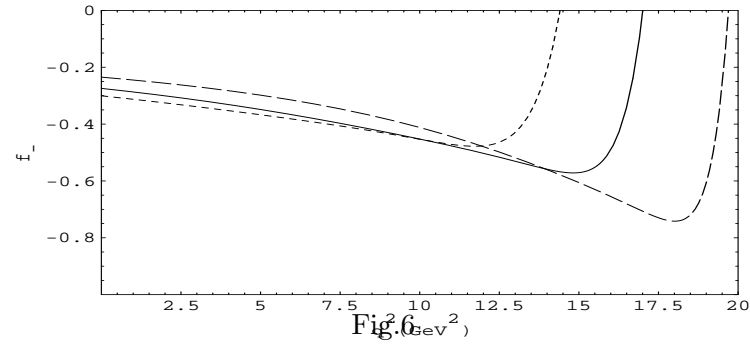
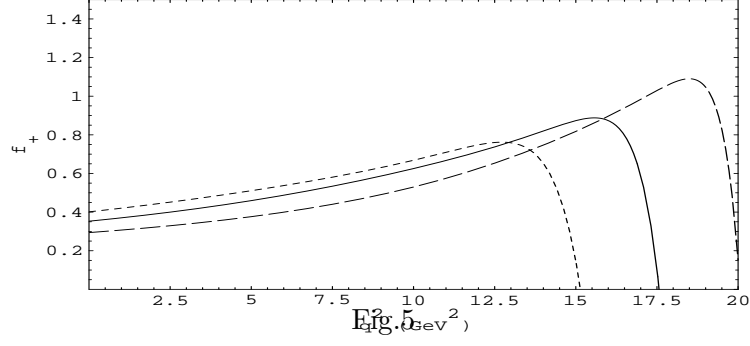


Fig.5-6. Variation of  $f_+$  and  $f_-$  with respect to the momentum transfer  $q^2$  for different values of the continuum threshold  $s_0$ . The dashed, solid and dotted curves correspond to  $s_0 = 1.7, 2.3$  and  $2.9$  GeV respectively.  $T=2.0$  GeV is fixed.

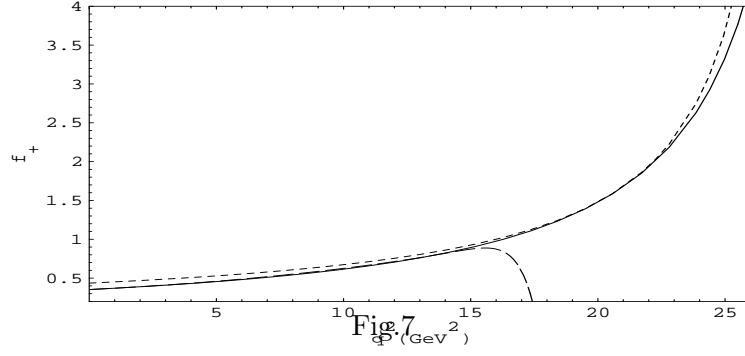


Fig.7. Variation of  $f_+$  from different estimations. The dashed curve is calculated from the sum rules (3.14) for  $s_0 = 2.3\text{GeV}$  and  $T = 2.0\text{GeV}$ . The dotted curve comes from the single pole model (4.4). And the solid curve is the result of (4.5), which we used to evaluate the integrated decay width and  $|V_{ub}|$ . The dashed line and the solid line almost overlap each other at  $q^2 < 15\text{GeV}^2$ .